Reg. No. :

Question Paper Code : 42768

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fourth Semester

Electronics and Communication Engineering

MA 2261 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulations 2008)

Time : Three Hours | Maximum : 100 Marks

Use of Statistical Tables is Permitted

Answer ALL questions

PART – A (10×2=20 Marks)

1. If a random variable X has m.g.f Mx(t) = 3/3-t, then find the standard deviation of X.

2. If X and Y are independent random variable with variance 2 and 3, then find the variance of 3X + 4Y.

3. Let X and Y have the joint p.m.f.

Y \ X	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

Find P(X + Y > 1).

4. If Z = aX + bY and r is the correlation coefficient between X and Y, then show that $\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2abr\sigma_x\sigma_y$.

5. For the sine wave process $X(t) = Y\cos\omega_0 t, -\infty < t < \infty$, ω_0 a constant the amplitude Y is a random variable with uniform distribution in the interval (0,1). Check whether the process is stationary or not.

6. Given that the auto correlation function for a stationary ergodic process with no periodic component is $R_{xx}(\tau) = 25 + 4 / (1 + 6\tau^2)$. Find the mean value and variance of the process $\{X(t)\}$.

7. Check whether the function $1/1+4\tau^2$ is valid auto correlation functions.

8. Define power spectral density function of a stationary process.

9. Define a linear system.

10. Define a time invariant system.

PART – B **(5×16=80 Marks)**

11. a) 1) A continuous random variable X has p.d.f. $f(x) = kx^2e^{-x}$, $x \geq 0$. Find k, r^{th} raw moment, mean and variance. **(8)**

2) Let X be a random variable with uniform distribution in the interval (– a, a). Determine 'a' so that $P(-1 \leq X \leq 2) = 0.75$ and $P(|X| < 1) = P(|X| > 2)$. **(8)**

(OR)

b) 1) The amount of time that a watch can run without having to be reset is a random variable having exponential distribution with mean 120 days. Find the probability that such a watch will have to be reset in less than 24 days. Not have to be reset for at least 180 days. **(8)**

2) If X is the exponential distribution given by $f(x) = e^{-x}$ for $x > 0$ and zero otherwise, then find the probability density of $Y = \sqrt{X}$ and $Y = X^2$. **(8)**

12. a) 1) Find the correlation between X and Y, if the joint probability density of X and Y is $f(x, y) = 2$ for $x > 0$, $y > 0$, $x + y < 1$ and zero otherwise. **(8)**

2) Calculate the rank coefficient of correlation for the following data : **(8)**

X :	68	64	75	50	64	80	75	40	55	64
Y :	62	58	68	45	81	60	68	48	50	70

(OR)

b) 1) The joint p.d.f. of a two-dimensional random variable (X, Y) is given by $f(x, y) = 4xy\, e^{-(x^2+y^2)}$, $x \geq 0$ and $y \geq 0$ and zero otherwise. Find the probability density function of $U = (X^2 + Y^2)^{1/2}$. **(8)**

2) The joint p.d.f. of the R.V(X, Y) is given by
$f(x, y) = x(1 + 3y^2)/4, 0<x<2$ and $0< y<1$. Find the marginal density function of X and Y, conditional density of X given Y and
$P(1/4 < X < 1/2/Y = 1/3)$. **(8)**

13. a) 1) Consider a random process {X(t)} defined by X(t) = U cost + V sin t when U and V are independent random variable each of which assumes the values – 2 and 1 with probabilities 1/3 and 2/3 respectively. Show that {X(t)} is wide sense stationary and not strict sense stationary. **(8)**

2) Define random telegraph process. Prove that it is stationary in the wide sense. **(8)**

(OR)

b) 1) Suppose that X(t) is a random telegraph signal process with E[X(t)] = 0 and $R(\tau) = e^{-2\lambda|\tau|}$. Find mean and variance of the time average of X(t) over $(-\tau\ \tau)$. Is it mean ergodic ? **(8)**

2) Suppose X(t) is a normal process with mean $\mu(t) = 3$ and
$C(t_1, t_2) = 4\, e^{-0.2}\,(|t_1 - t_2|)$. Find the probability that $X(5) \leq 2$
and $|X(8) - X(5)| \leq 1$. **(8)**

14. a) 1) If the process {X(t)} is defined as X(t) = Y(t) Z(t), where {Y(t)} and {Z(t)} are independent WSS process, prove that $R_{XX}(\tau) = R_{YY}(\tau)\, R_{ZZ}(\tau)$ and
$S_{XX}(w) = (1/2\,\pi) \int_{-\infty}^{\infty} S_{YY}(\alpha) S_{ZZ}(w-\alpha)\, d\alpha$. **(8)**

2) Show that the spectral density function of a real random process is an even function. **(8)**

(OR)

b) 1) The power spectral density function of a zero mean WSS process {X(t)} is given by S(w) = 1, $|w| < w_0$ and zero, otherwise. Find $R(\tau)$. Show also that

X(t) and $X\left(r + \dfrac{\tau}{w_0}\right)$ are uncorrelated. **(8)**

2) Define cross correlation function and write its 4 properties. **(8)**

15. a) If {X(t)} is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)\,(x(t-u))du$, then
$Ryy(\tau) = Rxx(\tau) * K(\tau)$, where $K(\tau)\ h(t) * h(-t) = \int_{-\infty}^{\infty} h(u) h((t+u))du$. **(16)**

(OR)

b) If the input to a time invariant stable linear system is a WSS process, then show that the output will also be a WSS process. **(16)**